# RAY OPTICS

## REFLECTION :

Light is a form of energy which is propagated as electromagnetic waves. It does not require a medium for its propagation. Its speed in free space (i.e., vacuum) is $3 \times 10^8$ m/s.

As in visible region the possible number of different wavelengths (or frequencies) between 4000 Å and 7000 Å are infinite and different frequencies produce the sensation of different colours, the number of colours in visible region is infinite. However, our eye can distinguish only following six colours.

Eye is most sensitive to yellow-green light, light of wavelength 5550 Å.

Persistence of eye is 1/10 s, i.e., if time interval between two successive light pulses is lesser than 0.1 s, eye cannot distinguish them separately. Also the resolving limit of the eye is one minute, two objects separated by distance d will not be distinctly visible to the eye if the angle, $\theta$ subtended by them at the eye,

$$0 < 0°1' \quad \text{i.e.,} \quad \frac{d}{D} < \frac{\pi}{(180 \times 60)}$$

When light passes from one medium to the other, velocity and wavelength m change, amplitude may decrease or remain constant, but frequency and colour of light do not change, i.e., colour of light is determined by its frequency (a not wavelength), e.g., if red light passes from air to water (or glass) its velocity and wavelength in water (or glass) will be different from that in air frequency and colour remain the same.

If the velocity of light in a medium (such as water or glass) is same in all directions, the medium is called isotropic. However, if the velocity of light is different in different directions (e.g., in calcite or quartz, etc.), the medium is said to be anisotropic for that light.

### Principle of Reversibility of Light:

If a light ray is reversed, it always retraces its path. Object and image positions are interchangeable. The points corresponding to object and image are called conjugate points.

### Optical Path :

It is defined as distance travelled by light in vacuum in the same time in which it travels a given path length in a medium. If light travels a path length d in a medium at speed v, the time taken by it will be (d/v). So optical path length.

$$L = c \times \left[\frac{d}{v}\right] = \mu d\,; \quad \left[\text{as} \frac{c}{v} = \mu\right]$$

As for all media $\mu > 1$, optical path length is always greater than actual path length.

## REFLECTION:

### Laws of Reflection :

The incident-ray, reflected-ray and normal to the reflecting surface at the point of incidence all lie in the same plane. The angle of reflection is equal to the angle of incidence, i.e., $\angle i = \angle r$.

## Reflection from Plane-surfaces

The image is always erect, virtual and of same size as the object. It is at the same distance behind the mirror as the object is in front of it.

- Some points regarding plane reflecting surface.
- If the rays after refraction or reflection actually converge at a point, the image's said to be real and it is not only to be seen but also obtained on a suitably placed screen. However, if the rays do not actually converge but appear to do so and actually diverge, the image is said to be virtual. A virtual image can only be seen and cannot be obtained on a screen at the position of the image.
- If an object moves towards a plane mirror at speed v, the image will also approach (or recede) at same speed v, the speed of image relative to object will be v – (–v) = 2v. Similarly if the mirror is moved towards or (away from) the object with speed v the image will move towards (or away from) the object with speed 2v.
- Deviation $\delta$ is defined as the angle between directions of incident ray and emergent ray. So if light is incident at an angle of incidence i, $\delta = 180 - (\angle i + \angle r) = (180 - 2i)$

  If keeping the incident ray fixed, the mirror is rotated by an angle $\theta$, about an axis in the plane of mirror, the reflected ray is rotated through an angle $2\theta$.
- The image formed by a plane mirror suffers lateral-inversion in the image formed by a plane mirror left is turned into right and vice-versa with respect to object as.
- To see his full image in a plane mirror a person requires a mirror of at least half of his height.
- To see a complete wall behind himself a person requires a mirror of at least (1/3) the height of wall and he must be in the middle of wall and mirror.
- If there are two plane mirrors inclined to each other at an angle $\theta$, the number of images of a point object formed are determined as follows:
- If $(360/\theta)$ is even integer (say m) number of images formed

  n = (m – 1), for all positions of object
- If $(360/\theta)$ is odd integer (say m) number of images formed

  n = m, if the object is not on the bisector of mirrors

  n = (m – 1), if the object is on the bisector of mirrors
- If $(360/\theta)$ is a fraction, the number of images formed will be equal to its integral part.

## Reflection at Spherical Surface

- The focal length of a spherical mirror of radius R is given by f = (R/2)
- The power of a mirror is defined as $P = -\frac{1}{f(\text{in m})} = -\frac{100}{f(\text{in cm})}$
- If a thin object of linear size O is situated vertically on the axis of a mirror at a distance u from the pole and its image of size I is formed at a distance v (from the pole), magnification (transverse) is defined as :

$$m = \left[\frac{I}{O}\right] = -\left[\frac{v}{u}\right]$$

–ve magnification implies that image is inverted with respect to object* +ve magnification means that image is erect with respect to object.

- If an object is placed at a distance u from the pole of a mirror and its image is formed at a distance v

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

- If the 1-D object is placed with its length along the principle axis, the so called longitudinal magnification becomes

$$m_L=\frac{I}{O}=-\frac{(v_2-v_1)}{(u_2-u_1)}=-\frac{dv}{du}=\left[\frac{v}{u}\right]^2=m^2$$

## NEWTON'S FORMULA:

- In case of spherical mirrors if object distance ($x_1$) and image distance ($x_2$) are measured from focus instead of pole u (f + $x_1$) and v = (f + $x_2$), the mirror formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \text{ reduces to } \frac{1}{(f+x_2)}+\frac{1}{(f+x_1)}=\frac{1}{f}$$

which on simplification gives $x_1 x_2 = f^2$

In case of spherical mirrors if we plot a graph between.

(1/u) and (1/v), it will be a straight line with intercept (1/f) with each axis as $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ becomes y + x = c with $c=\frac{1}{f}$.

Graph between u and v will be a hyperbola, as for $u = f, v=\infty$ and for $u=\infty$, $v = f$. A line u = v will cut this hyperbola at (2f, 2f).

Concave mirror behaves as convex lens (both convergent) while convex mirror behaves as concave lens (both divergent).

## USASGE OF CONVEX / CONCAVE MIRRORS:

As convex mirror gives erect, virtual and diminished image. In convex mirror, the field of view is increased as compared to plane mirror. This is why it is used as rear-view mirror in vehicles. Concave mirrors give enlarged, erect and virtual image (if object is between F and P), so these are used by dentists for examining teeth. Further due to their converging property concave mirrors are also used as reflectors in automobile head lights and search lights and by ENT surgeons in ophthalmoscope.

## REFRACTION :

### Law of Refraction:

1. Incident ray, Refracted ray and normal at a point lie in same plane.
2. Product of refractive index and the sine of angle made by light ray with the normal remains constant when light travels from one medium to another.

i.e. $\mu \times \sin i = \text{constant}$

$$\mu_1 \times \sin i = \mu_2 \sin r$$

If light passes from rarer to denser medium $\mu_1=\mu_R$ and $\mu_2=\mu_D$ so that in passing from rarer to denser medium, the ray bends towards the normal.

If light passes from denser to rarer medium $\frac{\sin i}{\sin r} = \frac{\mu_R}{\mu_D} < 1$

in passing from denser to rarer medium, the ray bends away from the normal.

## Apparent Depth and Normal Shift

$$\frac{d_{Ac}}{d_{Ap}} = \frac{\mu_1}{\mu_2}$$

Object in a denser medium is seen from a rerer medium.

The distance between object and its image, called normal shift and with $d_{Ac} = t$, will be

$$x = d_{Ac} - d_{Ap} = t - (t/\mu) = t[1 - (1/\mu)]$$

Object in a rarer medium is seen from a denser medium

$$x = d_{Ap} - d_{Ac} = [(\mu - 1)]t$$

Further in passing through a medium of thickness t and refractive index $\mu$, a ray incident at a small angle $\theta$ is displaced parallel to itself by 'y' called lateral displacement.

$$y = \left[\frac{\mu - 1}{\mu}\right]t$$

If there are number of liquids of different depths, one over the other

$$d_{Ac} = d_1 + d_2 + d_3 \ldots \quad \text{and} \quad d_{Ap} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \frac{d_3}{\mu_3} + \ldots$$

$$\mu = \frac{d_{Ac}}{d_{Ap}} = \frac{d_1 + d_2 + \ldots}{(d_1/\mu_1) + (d_2/\mu_2) + \ldots} = \frac{\Sigma d_i}{\Sigma(d_i/\mu_i)}$$

$$\mu = \left[\frac{2\mu_1\mu_2}{\mu_1 + \mu_2}\right] = \text{Harmonic mean.}$$

## Total Internal Reflection :

If light is passing from denser to rarer medium through a plane boundary, then $\mu_1 = \mu_D$ and $\mu_2 = \mu_R$; so with $\mu = (\mu_D/\mu_R)$,

$$\sin i = \frac{\mu_R}{\mu_D}\sin r \quad \text{i.e.,} \quad \sin i = \frac{\sin r}{\mu}$$

$$\sin i \propto \sin r \quad \text{with} \quad (\angle i) < (\angle r)$$

So as angle of incidence i increases angle of refraction r will also increase and for certain value of i (<90°) r will become 90°. The value of angle of incidence for which r = 90° is called critical angle and is denoted by $\theta_C$ and in the light will be given by

$$\sin\theta_C = \frac{\sin 90}{\mu} \quad \text{i.e.,} \quad \sin\theta_C = \frac{1}{\mu}$$

the total light incident on the boundary will be reflected back into the same medium from the boundary. This phenomenon is called total internal reflection. Here it is worthy to note that :

For total internal reflection to take place light must be propagating from denser to rarer medium.

In case of total internal reflection, as all (i.e., 100%) incident light is reflected back into the same medium there is no loss of intensity while in case of reflection from mirrors or refraction from lenses there is some loss of intensity as all light can never be reflected or refracted.

From Snell's law, $\theta_C = \sin^{-1}(1/\mu)$

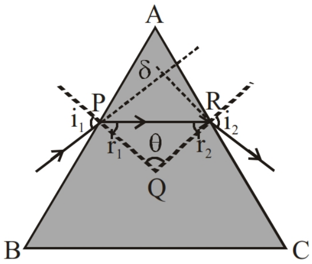
## PRISM-THEORY:

Prism is a transparent medium bounded by any number of surfaces in such a way that the surface on which light is incident and the surface from which light emerges are plane and non-parallel.

Angle of prism or refracting angle of prism means the angle between the faces on which light is incident and from which it emerges.

Angle of deviation means the angle between emergent and incident rays.

The angle of deviation $\delta$ will be




$$\delta = (i_1 - r_1) + (i_2 - r_2) \qquad \left[ r_1 + r_2 + \theta = 180^o \Rightarrow A + 90^o + \theta + 90^o = 360 \Rightarrow A + \theta = 180^o \right.$$

$$\delta = (i_1 + i_2) - (r_1 + r_2) \qquad \left. r_1 + r_2 + \theta = A + \theta \Rightarrow r_1 + r_2 = A \right]$$

$$\delta = [i_1 + i_2 - A] \qquad (r_1 + r_2) = A$$

This is the required result and holds good if emergent ray exists.

If angle of prism A is small, $r_1$ and $r_2$ (as $r_1 + r_2 = A$) and hence $i_1$ and $i_2$ will also be small. Since for small angles sin $\theta = \theta$. Snell's law at first and second surfaces of prism gives respectively:

$$i_1 = \mu r_1 \quad \text{and} \quad \mu r_2 = i_2$$

$$(i_1 + i_2) = \mu(r_1 + r_2) = \mu A$$

So for small angle of prism.

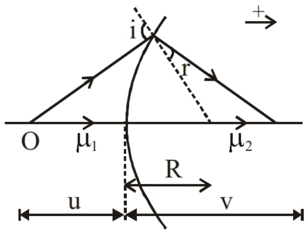
Deviation will be maximum when angle of incidence $i_1$ is maximum

$$\delta_{max} = 90 + i_2 - A$$

## MINIMUM DEVIATION :

Minimum deviation of a ray through the prism is given by: $\mu = \frac{\sin[(A+\delta_m)/2]}{\sin(A/2)}$

## Refraction at Spherical Surfaces




Refraction at single spherical surface is shown in the figure above.

Here light rays starting form an object O, placed in a medium of R.I. $\mu_1$, move into another medium of R.I. $\mu_2$ after being refracted at a curved surface of radius R. Object distances (u) and image distance (v) as shown in the figure, are measured from pole. Radius of curvature (R) is also measured from the pole, as shown. Then

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

For refraction at single surface and for real objects,

$\mu$ is always –ve

v is +ve if image forms on otherside, ie in next medium.

v is –ve if image forms on same side, ie in first medium

R is +ve if surface is convex as seen from the object

The image is real if formed inside the second medium, because such image is due to actual meeting of rays after refraction. If formed on the same side, there is no actual meeting of rays, hence the image is virtual.

Lateral magnification, $m = \frac{\text{Image size}}{\text{Object size}} = \frac{\mu_1 v}{\mu_2 v}$

## Refraction Through Lens

### Lens maker's formula for thin lens

Let $R_1$ and $R_2$ be radii of curvature of the first and second spherical surface. Let f be the focal length of the lens and $\mu$ be the refractive index of lens material w.r.t. the medium in which the lens is placed. Then

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{or} \quad P = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$ where P is power of the lens

### Linear Magnification for a lens

Magnification $m = \frac{v}{u}$

We know that $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ or $\frac{v}{v} - \frac{v}{u} = \frac{v}{f}$ $\Rightarrow 1 - \frac{v}{u} = \frac{v}{f}$ $\Rightarrow \frac{v}{u} = \frac{f - v}{f}$ $\Rightarrow m = \frac{f - v}{f}$

We can also express m in terms of u and f.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \text{or} \quad \frac{u}{v}-\frac{u}{u}=\frac{u}{f} \quad \Rightarrow \quad \frac{u}{v}=\frac{u+f}{f} \quad \Rightarrow \quad m=\frac{f}{f+u}$$

## Power of a Lens

$$P=\frac{1}{f}$$

Since focal length of a convex lens or a converging lens is positive, therefore its power is positive. Similarly, the power of a concave lens or a diverging lens is negative.

Opticians express the power of a lens in terms of a unit called the DIPOTRE. It is regarded as the SI unit of optical power. The power of a lens is said to be one dioptre if the focal length of the lens is 1 metre.

When focal length is in cm, $P=\frac{100}{f}$ dioptre.

## Lens Formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

This equation holds good for both convex and concave lenses, whether the image formed is real or virtual.

For real image, v is +ve because real image always forms or other side of the lens. For virtual image v is –ve.

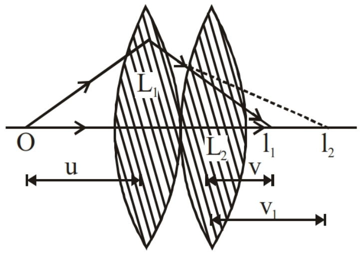
Focal length f is +ve for convex lens and –ve for concave lens. Object distance u is always -ve for a real object and u is always +ve for a virtual object.

Convex Lens

| Position of object | Position of image | Real/Virtual | Inverted/erect | Magnification and size of image |
|---|---|---|---|---|
| at infinity | at focus | real | inverted | $m<1$ greatly diminished |
| beyond 2f | between f and 2f | real | inverted | $m=1$ same size |
| at 2f | at 2f | real | inverted | $m=1$ same size |
| between f and 2f | beyond 2f | real | inverted | $m>1$ magnified |
| at f | at inifinity | real | inverted | $m=\infty$ magnified |
| between optical centre and focus | at a distance greater than the object distance and on the same side object | virtual | erect | $m>1$ magnified |
| Concave Lens | | | | |
| at inifinity | at focus ($v=f$) | virtual | erect | $m<1$ diminished |
| between infinity and optical centre | between optical centre and focus | virtual | erect | $m<1$ diminished |

## Focal Length of Combination of two thin lenses in contact

Let two lenses be in contact, as shown in the figure. Focal length of the combination is given by




$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$

If P is the power of the combination, then $P = P_1 + P_2$ where $P_1$ and $P_2$ are the powers of the individual lenses. If two lenses are separated by distance d then focal length of system of two lenses is given by

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$$

## Displacement Method for finding the focal length of a convex lens

In this method, the distance between the object and the screen must be greater than 4f, where f is the focal length of the convex lens. The image on the screen can be formed corresponding to two different positions of the lens. Figure (i) shows the magnified image of size $I_1$ for the position $L_1$ of the lens.

$$m_1=\frac{I_1}{O}=\frac{v}{u}$$

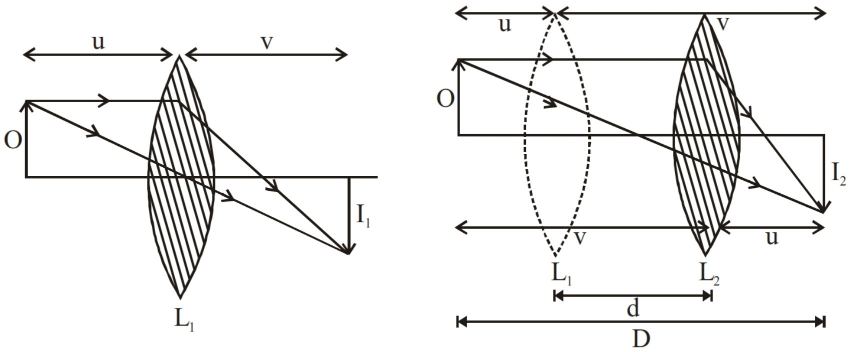



Figure (ii) shows the diminished image of size $I_2$ for the position $L_2$ of the lens.

$$m_2=\frac{I_2}{O}=\frac{v}{u}$$

From (1) and (2),

$$\frac{I_1 I_2}{O^2}=\frac{v}{u}\times\frac{u}{v}=1$$

or $\quad O=\sqrt{I_1 I_2} \Rightarrow \text{object size} = \sqrt{(\text{1st image size})\times(\text{2nd image size})}$